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Program and Abstracts

Viscous Heating in Nonisothermal Die Flows of Viscoplastic Fluids With Wall Slip

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INTRODUCTION

Analysis of the nonisothermal flows of generalized Newtonian fluids in simple dies has been carried-out by a number of investigators (1-6). Generally power-law behavior was assumed. However, many materials, including concentrated suspensions, elastomers, gels, food products and energetic compounds exhibit viscoplasticity, the flow and deformation behavior of which is generally subject to the occurrence of wall slip (7). Thus, realistic models for such materials should incorporate the wall slip boundary condition at the wall typically by using the velocity, u_S vs. the wall shear stress τ_W data, i.e., $u_s = \beta \tau_w$ where β is the Navier's wall slip coefficient. Here we present an analytical model of the flow of viscoplastic fluid flowing through simple (slit or cylindrical) dies under non-isothermal condition, subject to Navier's wall slip coefficient.

ANALYSIS (Velocity field)

The equation of conservation of momentum in dimensionless form for this one-dimensional flow is given by:

$$\frac{1}{\xi^p}\frac{\partial}{\partial \xi}\left(\xi^p\left|\frac{du_z^*}{d\xi}\right|^{n-1}\frac{du_z^*}{d\xi}\right) = -1 \pm \tau_o^*\frac{p}{\xi} \tag{1a}$$

where p = 0 and 1 for slit and tubular dies, respectively. The following dimensionless variables have been defined:

$$u_z^* = u_z \Big/ \left(\left|dP/dz\right|/m\right)^s r_o^{1+s};\ \xi = r/r_o;\ \tau_o^* = \tau_o / r_o(-dP/dz) \tag{1b}$$

where dP/dz is the pressure gradient, m the consistency index, s=1/n the inverse power-law exponent and τ_0 the yield stress are the parameters of the Herschel-Bulkley constitutive equation, and r_O is radius of the cylindrical die or the gap of the slit die.

Slit die

Case 1: Floating plug region

$$u_z^* = \frac{\lambda_1^{1+s}}{(s+1)} - \frac{(\lambda_1-\xi)^{1+s}}{(s+1)} + \kappa + \alpha\lambda_1 \qquad 0 \le \xi \le \lambda_1 \tag{2a}$$

$$u_z^* = \frac{(1-\lambda_2)^{1+s}}{(s+1)} - \kappa - \alpha(\lambda_1 - 1) \qquad \lambda_1 \le \xi \le \lambda_2 \tag{2b}$$

$$u_z^* = \frac{(1-\lambda_2)^{1+s}}{(s+1)} - \frac{(\xi-\lambda_2)^{1+s}}{(s+1)} - \kappa - \alpha(\lambda_1 - 1) \qquad \lambda_2 \le \xi \le 1 \tag{2c}$$

Cylindrical tube die

$$u_z^* = \frac{\alpha}{2} + \frac{(1-2\tau_o^*)^{1+s}}{2^s\ (s+1)} \qquad 0 \le \xi \le 2\tau_o^* \tag{3a}$$

$$u_z^* = \frac{\alpha}{2} + \frac{1}{2^s\ (s+1)}\left[(1-2\tau_o^*)^{1+s} - (\xi - 2\tau_o^*)^{1+s}\right] \qquad 2\tau_o^* \le \xi \le 1 \tag{3b}$$

where $\alpha=\beta r_o(-dP/dz)/(|dP/dz|/m)^s r_o^{1+s}$, $\kappa=\beta\tau_o/(|dP/dz|/m)^s r_o^{1+s}$ and λ_1 and λ_2 are extremum locations in the flow field.

Temperature field

In dimensionless form, the equation of energy and the boundary conditions are given by:

$$u_z^* \frac{\partial \theta}{\partial z^*} = \frac{1}{\xi^p} \frac{\partial}{\partial \xi}\left(\xi^p \frac{\partial \theta}{\partial \xi} \right) + G f(\xi) \tag{4a}$$

$$\theta = C(\xi) \;\; z^* = 0,\, 0 \le \xi \le 1; \frac{\partial \theta}{\partial \xi} = +Bi_o \theta \;\; z^* > 0,\, \xi = 0; \frac{\partial \theta}{\partial \xi} = -Bi_1 \theta \;\; z^* > 0, \xi = 1 \tag{4b}$$

where $\theta = (T - T^*)/\Delta T$; $z^* = z/r_o\, P\acute{e}$; $P\acute{e} = \rho C_p (|dP/dz|/m)^s r_o^{1+s} r_o / k$ and

$$G = (m/k\Delta T)\left(\left(|dP/dz|/m\right)^s r_o^s\right)^{n-1}\left(\left(|dP/dz|/m\right)^s r_o^{1+s}\right)^2 \tag{4c}$$

T^* is a reference temperature, ΔT is a reference temperature difference, Bi_o $(= h_o r_o/k)$, Bi_1 $(= h_1 r_o/k)$ are Biot moduli and $f(\xi)$ is the viscous dissipation function. To solve Eqs. 4a-b, we apply the finite integral transform technique(8), and the solution for θ is of the form:

$$\theta = \sum_{m=1}^{\infty} \frac{\psi_m}{\int_0^1 u_z^* \psi_m^2 \xi^p d\xi} \left[\frac{\left(\int_0^1 G f(\xi) \psi_m \xi^p d\xi\right)\left(1 - \exp(-\lambda_m^2 z^*)\right)}{\lambda_m^2} + \left(\int_0^1 u_z^* \psi_m C(\xi) \xi^p d\xi\right) \exp(-\lambda_m^2 z^*) \right] \tag{5}$$

RESULTS

Typical results for various values of β, τ_0 and G are shown in Figures 1-5. As the wall slip coefficient β increases, the temperature rise decreases since increased β results in a decreased velocity gradient. The isothermal wall condition (Fig. 2) produces a significantly lower temperature rise than the adiabatic condition (Fig. 1). Increase in yield stress decreases the velocity gradient but increases the viscous dissipation function(Fig. 3). The latter factor appears to predominate, resulting in increased temperature rise, which could be up to a factor of 2 or more, with increased yield stress. As the Griffith number G, increases so does the temperature rise(Fig. 4). For the adiabatic condition (Fig. 4), the maximum temperature rise occurs at the wall. Experimental bulk temperature rise data collected with a slit die (9) are compared with theoretical predictions in Fig. 5. The adiabatic and isothermal wall conditions appear to provide the bounds for actual temperature rise suggesting that a finite non-zero Biot value is needed for the thermal boundary condition.

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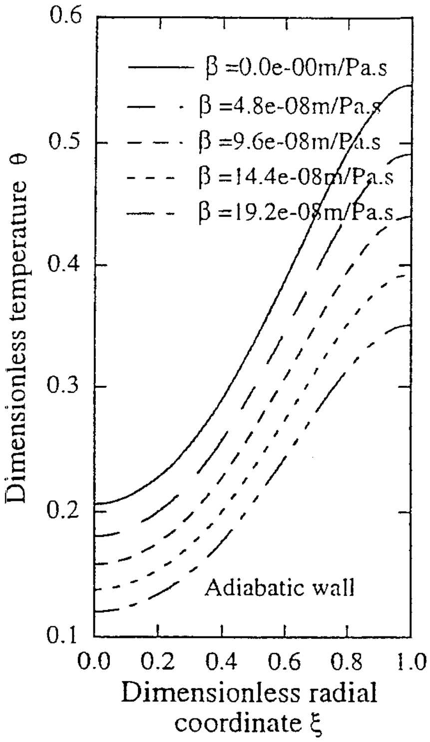

Fig. 1 Dimensionless radial temperature profiles in a cylindrical die with adiabatic wall for varying β

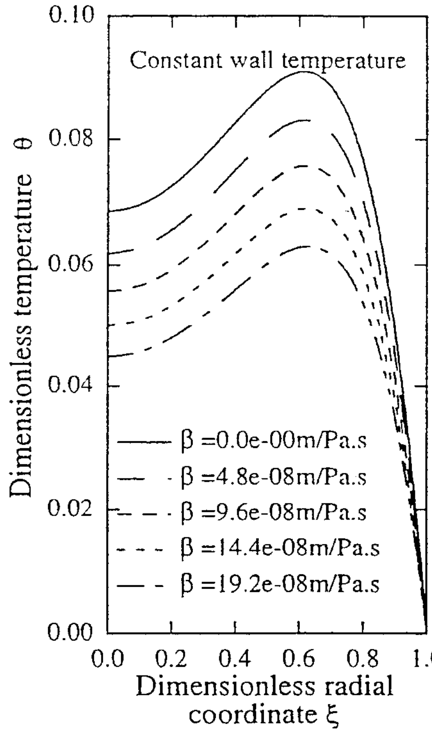

Fig. 2 Dimensionless radial temperature profiles in a cylindrical die with isothermal wall for varying β

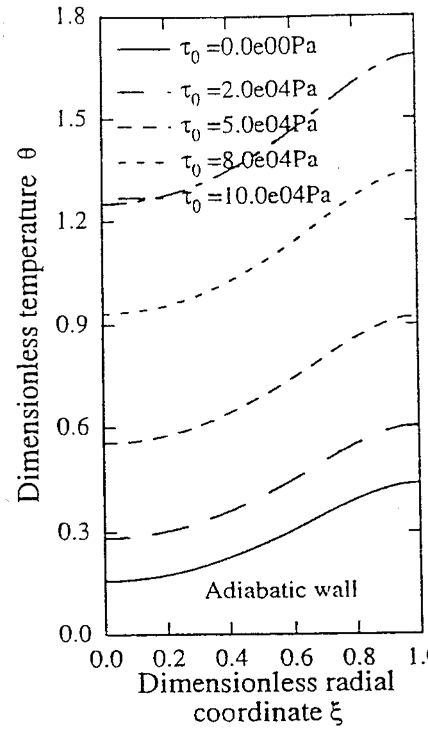

Fig. 3 Dimensionless radial temperature profiles in a cylindrical die with adiabatic wall for varying τ_0

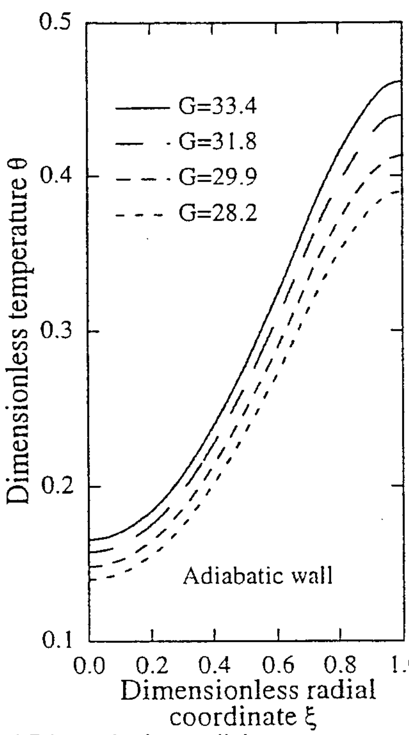

Fig. 4 Dimensionless radial temperature profiles in a cylindrical die with adiabatic wall for varying G

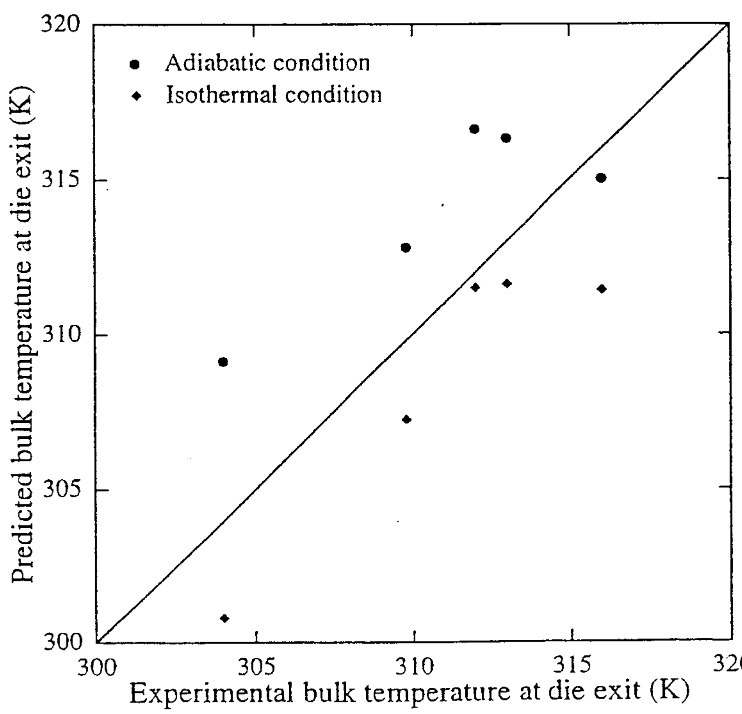

Fig. 5 Comparison of experimental temperature rise data in a slit die with theoretical predictions for a highly filled suspension